

Indian Statistical Institute
Supplementary Examination
Algebra I
2017-2018

Max Marks: 100

Time: 3 hours.

Answer all questions.

1. (a) Find all automorphisms of the cyclic group of order 10.
(b) Let \mathbb{C}^\times denote the group of non-zero complex numbers and $H = \{1, -1, i, -i\}$ be the subgroup of \mathbb{C}^\times of fourth roots of unity. Describe the cosets of H in \mathbb{C}^\times explicitly. Is \mathbb{C}^\times/H isomorphic to \mathbb{C}^\times ? Give reasons. (10+10)
2. (a) Show that two elements in S_n are conjugate if and only if they are of the same cycle type.
(b) Show that if n is odd then the set of all n -cycles consists of two conjugacy classes of equal size in A_n . (5+15)
3. (a) State Sylow's Theorems.
(b) Show that if G is a group of order p^2q , where p, q are distinct primes, then G is not simple. (6+14)
4. (a) Define commutator subgroup of a group G . Show that the commutator subgroup is a normal subgroup of G .
(b) Show that the commutator subgroup of S_n is A_n , for all $n \geq 3$. (6+14)
5. (a) Define (external) semidirect product of two groups G and H .
(b) Classify all groups of order 55. (6+14)